Applied Statistics and Mathematics in Economics & Business

BS1501

Tutorial 2

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(Champ)

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1. Mutually Exclusive?

(a) Being an economist and being a professor: 
   Not mutually exclusive

(b) Throwing a 5 or 6 with one die: 
   Mutually exclusive

(c) Being an M.P. and being 20 years of age. 
   (Minimum age of an M.P. is 21): 
   Mutually exclusive

(d) Drawing a red card or an ace out of a pack of cards: 
   Not mutually exclusive
2. Accountancy employees

(a) Prob. of employees from Wales: $P(W)$
   \[ = \frac{\text{from Wales}}{\text{All employees}} = \frac{6}{15} \]

(b) Prob. of employees Scotland: $P(S)$
   \[ = \frac{\text{from Scotland}}{\text{All employees}} = \frac{4}{15} \]
2. Accountancy employees

(c) Prob. of employees NOT from Wales:

= All prob. – P(W)
= 1 – P(W)
= 1 – (6/15) = 9/15
2. Accountancy employees

(d) Prob. of employees From Wales and Scotland

\[ P(W) + P(S) = \frac{6}{15} + \frac{4}{15} = \frac{10}{15} \]
3. Primary school in Cardiff

I = the event that the boy likes ice-cream
C = the event that the boy likes cake

\[ P(I) = 0.75, \quad P(C) = 0.55, \quad P(I \text{ and } C) = 0.40 \]

(a) \[ P(I \text{ or } C \text{ or both}) = P(I) + P(C) - P(I \text{ and } C) \]
\[ = 0.75 + 0.55 - 0.40 = 0.90 \]
3. Primary school in Cardiff

I = the event that the boy likes ice-cream
C = the event that the boy likes cake

\[ P(I) = 0.75, \ P(C) = 0.55, \ P(I \text{ and } C) = 0.40 \]

(b) Neither ice-cream nor cake

\[ = 1 - P(I \text{ or } C \text{ or Both}) \]
\[ = 1 - 0.90 \]
\[ = 0.1 \]
## 4. Accidents

<table>
<thead>
<tr>
<th></th>
<th>Accident</th>
<th>No accident</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 45 years</td>
<td>25</td>
<td>35</td>
</tr>
<tr>
<td>45 years or older</td>
<td>5</td>
<td>35</td>
</tr>
</tbody>
</table>
## 4. Accidents

<table>
<thead>
<tr>
<th></th>
<th>Accident</th>
<th>No accident</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Under 45 years</strong></td>
<td>25</td>
<td>35</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(25 + 35)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>45 years or older</strong></td>
<td>5</td>
<td>35</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5 + 35)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>30</td>
<td>70</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>(25 + 5)</td>
<td>(35 + 35)</td>
<td></td>
</tr>
</tbody>
</table>
### 4. Accidents

<table>
<thead>
<tr>
<th></th>
<th>Accident</th>
<th>No accident</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Under 45 years</strong></td>
<td>0.25 (25/100)</td>
<td>0.35 (35/100)</td>
<td>0.60 (60/100)</td>
</tr>
<tr>
<td><strong>45 years or older</strong></td>
<td>0.05 (5/100)</td>
<td>0.35 (35/100)</td>
<td>0.40 (40/100)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>0.30 (30/100)</td>
<td>0.70 (70/100)</td>
<td>1.0 (100/100)</td>
</tr>
</tbody>
</table>
4. Accidents

(b) \( P(\text{Accident} \mid \text{under 45}) \)

\[
= \frac{P(\text{Accident and under 45})}{P(\text{Under 45})}
= \frac{0.25}{0.60} = 0.416
\]

<table>
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<th>No accident</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 45 years</td>
<td>0.25 (25/100)</td>
<td>0.35 (35/100)</td>
<td>0.60 ([25+35]/100)</td>
</tr>
<tr>
<td>45 years or older</td>
<td>0.05 (5/100)</td>
<td>0.35 (35/100)</td>
<td>0.40 (40/100)</td>
</tr>
<tr>
<td>Total</td>
<td>0.30 (30/100)</td>
<td>0.70 (70/100)</td>
<td>1.0 (100/100)</td>
</tr>
</tbody>
</table>
4. Accidents

(c) $P(\text{Accident} \mid \text{over 45})$

\[
= \frac{P(\text{Accident and over 45})}{P(\text{over 45})}
= \frac{0.05}{0.40} = 0.125
\]

<table>
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<tbody>
<tr>
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<td>0.60 ([25+35]/100)</td>
</tr>
<tr>
<td></td>
<td>(25/100)</td>
<td>(35/100)</td>
<td></td>
</tr>
<tr>
<td><strong>45 years or older</strong></td>
<td><strong>0.05</strong></td>
<td>0.35</td>
<td>0.40 (40/100)</td>
</tr>
<tr>
<td></td>
<td><strong>(5/100)</strong></td>
<td>(35/100)</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>0.30</td>
<td>0.70</td>
<td>1.0 (100/100)</td>
</tr>
<tr>
<td></td>
<td>(30/100)</td>
<td>(70/100)</td>
<td></td>
</tr>
</tbody>
</table>
5. Economics course

\[ P(M) = P(F) = \frac{1}{2} \]
\[ P(E \mid M) = \frac{1}{6} \]
\[ P(E \mid F) = \frac{1}{30} \]

(a) Male and Economics
\[ P(M \text{ and } E) = P(E \mid M) \times P(M) \]
\[ = \left(\frac{1}{6}\right) \times \left(\frac{1}{2}\right) = \frac{1}{12} \]
5. Economics course

\[ P(M) = P(F) = \frac{1}{2} \]

\[ P(E | M) = \frac{1}{6} \]

\[ P(E | F) = \frac{1}{30} \]

\[ P(E) = 10\% = \frac{1}{10} \]

(b) \[ P(M \text{ or } E) = P(M) + P(E) - P(M \text{ and } E) \]

\[ = \frac{1}{2} + \frac{1}{10} - \frac{1}{12} \]

\[ = \frac{31}{60} \]
Combination:
Number of ways of selecting $x$ object from a set of $n$, when the order in unimportant

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

$$\binom{10}{4} = \frac{10!}{4!6!} = \frac{10 \times 9 \times 8 \times 7 \times 6!}{4 \times 3 \times 2 \times 1 \times 6!} = \frac{10 \times 9 \times 8 \times 7}{(4 \times 3 \times 2 \times 1)} = \frac{10 \times (3 \times 3) \times 8 \times 7}{(8 \times 3)}$$

$n = 10$
$x = 4$
$(n-x) = 6$

$$= \frac{10 \times 3 \times 7}{1} = 210$$
## 7. Company’s Demand

<table>
<thead>
<tr>
<th>Weekly unit Demand (x)</th>
<th>Probability: f(x)</th>
<th>E (x) = x*f(x)</th>
<th>x – E (x)</th>
<th>[x-E(x)]^2*f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>0.10</td>
<td>80.00</td>
<td>-185.00</td>
<td>3422.50</td>
</tr>
<tr>
<td>900</td>
<td>0.25</td>
<td>225.00</td>
<td>-85.00</td>
<td>1806.25</td>
</tr>
<tr>
<td>1000</td>
<td>0.40</td>
<td>400.00</td>
<td>15.00</td>
<td>90.00</td>
</tr>
<tr>
<td>1100</td>
<td>0.20</td>
<td>220.00</td>
<td>115.00</td>
<td>2645.00</td>
</tr>
<tr>
<td>1200</td>
<td>0.05</td>
<td>60.00</td>
<td>215.00</td>
<td>2311.25</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td><strong>Expected value</strong></td>
<td><strong>985.00</strong></td>
<td></td>
<td><strong>Variance</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>Std Deviation</strong></td>
</tr>
</tbody>
</table>
7. Company’s Demand

(a) \( E(x) = \text{Sum } [x \cdot f(x)] \)
\[
= 800(0.10) + 900(0.25) + \\
1000(0.40) + 1100(0.20) + 1200(0.05) \\
= 985
\]

(b) \( \text{Var}(x) = \text{Sum } [(x-E(x))^2 \cdot f(x)] \)
\[
= (800-985)^2(0.10) + (900-985)^2(0.25) + \\
(1000-985)^2(0.40) + (1100-985)^2(0.20) + \\
(1200-985)^2(0.05) \\
= 10,275
\]
Standard deviation = \( \sqrt{10,275} = 101.36 \)
8. Insurance Salesperson

\[ P(x) = \binom{n}{x} [p(x)]^x [1 - p(x)]^{n-x} \]

(a) \[ P(x = 6) = \binom{6}{6} (2/3)^6 (1/3)^0 = 0.088 \]

(b) \[ P(x = 4) = \binom{6}{4} (2/3)^4 (1/3)^2 = 0.329 \]
8. Insurance Salesperson

\[ P(x) = \binom{n}{x} [p(x)]^x [1 - p(x)]^{n-x} \]

\[
\binom{n}{x} = \frac{n!}{x!(n-x)!}
\]

(c) \[ P(x > 1) = 1 - [ P(x = 0) + P(x = 1) ] \]

\[
P(x = 0) = \binom{6}{0}(2/3)^0(1/3)^6
\]

\[
P(x = 1) = \binom{6}{1}(2/3)^1(1/3)^5
\]

\[ = 1 - 0.02 \]

\[ = 0.98 \]

(d) \[ \mu = np = 6(2/3) = 4 \]
9. Fatal Rate

**Poisson Distribution:**
Estimating the number of occurrence of an event within specific interval of time

A) No death in a one year period

\[ x = 0 \]

\[ \lambda(1\text{ year}) = 1.5 \]

\[
P(x) = \frac{\lambda^x e^{-\lambda}}{x!}
\]

\[
P(x = 0) = \frac{\lambda^0 e^{-\lambda}}{0!} = \frac{1.5^0 e^{-1.5}}{0!} = 0.2231
\]
9. Fatal Rate

**Poisson Distribution:**
Estimating the number of occurrence of an event within specific interval of time

b) No death in 6 months period
\[ x = 0 \]
\[ \lambda(6\text{months}) = \frac{1.5}{2} = 0.75 \]

\[ P(x) = \frac{\lambda^x e^{-\lambda}}{x!} \]
\[ P(x = 0) = \frac{0.75^0 e^{-0.75}}{0!} = 0.4724 \]
9. Fatal Rate

c) \( P(\geq 2 \text{ accidents in 6 months}) \)

\[
\lambda(6\text{months}) = 0.75
\]

\[
= 1 - P(x=0) - P(x=1)
\]

\[
= 1 - 0.4724 - 0.3543
\]

\[
= 0.1733
\]
10. Train (Cardiff – London)

(a) \( x \sim N(\mu = 120, \sigma = 10) \) \hspace{1cm} P(x > x_0) = P\left(Z > \frac{x_0 - \mu}{\sigma}\right)

\[
P(x > 130) = P\left(Z \geq \frac{130 - 120}{10}\right) = P(Z > +1) = 0.1586
\]
10. Train

\[(b) \quad x \sim N(\mu = 120, \sigma = 10) \quad P(x > x_0) = p\left(Z \geq \frac{x_0 - \mu}{\sigma}\right)\]

\[P(110 < x < 125) = P\left(\frac{110 - 120}{10} < Z < \frac{125 - 120}{10}\right)\]

\[= P(-1 < Z < +0.5)\]
10. Train

(b) \( P(-1 < Z < +0.5) \)

\[
= 1 - [P(Z<-1) + P(Z>0.5)]
\]

\[
P(Z < -1) = P(Z > 1)
\]

\[
= 1 - [P(Z>1) + P(Z>0.5)]
\]

\[
= 1 - 0.15866 - 0.30854
\]

\[
= 0.5328
\]
10. Train

(c) \( x \sim N(\mu = 120, \sigma = 10) \)

\[
P(x > x_0) = p\left(Z \geq \frac{x_0 - \mu}{\sigma}\right)
\]

\[
P(x > x_0) = p\left(Z \geq \frac{x_0 - 120}{10}\right)
\]

\[P(x > x_0) = 10\% = 0.10\]

\[0.10 = p\left(Z \geq \frac{x_0 - 120}{10}\right)\]
\[ 0.10 = p\left( Z \geq \frac{x_0 - 120}{10} \right) \]

\[ 1.2816 = \left( \frac{x_0 - 120}{10} \right) \]

\[ X_0 = 132.816 \text{ minutes} \]
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  me@pairach.com